

Job search with ubiquity and the wage distribution¹

Bruno Decreuse²

Université de la Méditerranée, GREQAM, IDEP

André Zylberberg³

Université Paris I Panthéon-Sorbonne, CNRS, CES

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²GREQAM, 2 rue de la charité, 13236 Marseille cedex 02, France. E-mail: decreuse@univmed.fr

³CES-EUREQua, Maison des Sciences Economiques, 106-112 boulevard de l'Hôpital, 75647 Paris cedex 13, France. E-mail: azyl@univ-paris1.fr

Abstract: We propose a search equilibrium model in which homogeneous firms post wages along with a vacancy to attract job-seekers, while homogeneous unemployed workers invest in costly job-seeking. The key innovation relies on the organization of the search market and the search behavior of the job-seekers. The search market is segmented by wage level, and individuals are ubiquitous in the sense they can choose the amount of search effort spent on *each* (sub-)market. We show that there exists a non-degenerate equilibrium wage distribution. Remarkably, the density of this wage distribution is hump-shaped, and it can be right-tailed. Our results are illustrated by an example originating a Beta wage distribution.

Keywords: Search effort; Segmented markets; Equilibrium wage dispersion

J.E.L. classification: D83; J31; J41; J64

1 Introduction

This paper is a theoretical contribution to the literature on frictional wage dispersion. We propose a search equilibrium model in which homogeneous firms post wages along with a vacancy to attract job-seekers, while homogeneous unemployed workers invest in costly job-seeking. The key innovation resides in the organization of the search market and the search behavior of the job-seekers. The search market is segmented by wage, and individuals are ubiquitous: they can choose the amount of search effort spent on *each* (sub-)market. We show that there exists a non-degenerate equilibrium wage distribution. The density of this wage distribution is hump-shaped, and it can be right-tailed. Our results are illustrated by an example originating a Beta wage distribution.

Why is this important? The size of residual wage dispersion in Mincerian wage regressions has motivated a major interest in understanding of wage distribution for homogeneous labor. Search frictions are natural ingredients behind the failure of the law of one price. Hence, modern theories of wage dispersion provide search-theoretic microfoundations for residual wage disparity. However, models of endogenous residual wage dispersion are more credible if they are able to reproduce the properties of empirical wage distributions. Policy analysts can be more confident in the predictions of such models regarding the impacts of unemployment compensation, payroll taxes, and so on. Labor economists are aware of this, and that is why models of frictional wage distribution have been estimated on a number of occasions. As we discuss below, the most commonly used models do not feature empirically plausible wage distributions. In particular, they do not predict the single-peakedness property. This particularity is usually pinned down by the explicit modelling of heterogeneity. Our model claims that search frictions alone can originate the single peak and the long right tail. This strengthens the point made by this particular field of Labor Economics. As a by-product, it also means that our model is a natural candidate for structural estimation.

Our paper is based on two key assumptions. First, the search market is segmented by wages. Jobs advertise wages, and two jobs paying different wages belong to two different search locations. Second, workers can choose their search investment on each market place. The technology that transforms search investment into probability to match has marginal decreasing returns. Workers can offset a lower return at given search intensity by a lower search intensity. This leads them to participate in a continuum of markets: they are, in this respect, *ubiquitous*.

In this setting, we obtain (i) a non-degenerate wage distribution, and (ii) a single-peaked density of the wage distribution as natural outcomes.

The main results can be explained as follows. Workers rationally seek jobs offering different wages, and, therefore, firms offer different wages in equilibrium. The path of market-specific search investment reflects the path of market-specific return to search. As the return to search first increases, and then decreases with wage, search investment evolves non-monotonously with the wage. In turn, the number of vacancies on each market responds to two effects. First, it tends to decrease with the wage, as paying higher wages must be compensated by lower search costs, and thus longer job queues. Second, it tends to increase with the number of effective job-seekers, because recruitment rates depend on the ratio of vacancies to effective number of job-seekers. As a result, the number of vacancies tends to adopt the path of market-specific search investment. The combination of these two effects implies that the number of vacancies is first increasing, and then decreasing in wage. It follows that the density of the wage offer

distribution is hump-shaped. Finally, the actual wage distribution can be deduced from the wage offer distribution and the knowledge of search investments. Its density is also single-peaked.

Equilibrium wage distributions are also consistent with another property of empirical wage distributions: they are right-tailed. In our framework, we define a long right tail by the requirement that the slope of the density of the distribution tends to zero as wage becomes closer to the upper bound of the support of the distribution. Then, we show that the density of the wage offer distribution is always right-tailed, while the density of the actual wage distribution may or may not be right-tailed. All these properties are illustrated by an example, in which the matching technology is Cobb-Douglas, and the efficiency of search effort is isoelastic. In that case, the wage offer distribution and the actual wage offer distribution follow Beta distributions.

Our paper matches two distinct ideas that have been investigated previously in the literature on search unemployment: search is directed, and individuals participate in different markets simultaneously.

First, search is directed: the search market is segmented by wages, and individuals choose which wage/job to prospect. Directed search models have been introduced first by Hosios (1990), Montgomery (1991), and Moen (1997). In such models, workers can choose which jobs they apply for – or, alternatively, which market they prospect –, while the probability of getting a job is a decreasing function of the length of the job queue. Wage competition thus takes place at the time of wage/market choice. In equilibrium, all wage offers must yield the same utility: if not, the jobs would not be prospected. This implies that the employment (recruitment) probability is a decreasing (an increasing) function of the wage. Typically, there is a unique wage offer balancing workers' marginal cost of seeking the highest wage offer (a lower employment probability) and their marginal benefit (a better wage once employed). Our model departs from usual directed search models because it features equilibrium wage dispersion. Interestingly, search investment is highest on the unique market that would have been prospected under directed search assumption.

Second, individuals simultaneously take part in different segments of the search market. This idea is increasingly popular in models devoted to two-sided heterogeneity. The search market is segmented by job type, and workers choose the subset of sub-markets they participate in. In models devoted to overeducation, educated workers seek both complex and simple positions while uneducated workers only search for simple jobs (see Gautier, 2002). In models with multidimensional skills, workers have a bundle of skills and participate in sub-markets on the basis of comparative advantage (see Moscarini, 2001), or on the basis of their ability to perform on the underlying technologies (see Charlot, Decreuse and Granier, 2005). Wage segmentation is a natural extension of job segmentation: different jobs are usually associated with different wages, so that individuals actually perceive job segmentation as wage segmentation. It is worth discussing the impact of this assumption in terms of congestion externalities. In our model, vacancies offering different wages do not create congestion effects on each other. An additional offer at 40,000 euro a year does not reduce the probability of filling a position offering 30,000 euro a year at given search intensities. However, the former offer raises the welfare of the unemployed. In response, the unemployed reduce their effort to obtain the latter wage offer. As a consequence, the probability to fill the latter position is lower. This argument may seem rather difficult to accept when one imagines the case of a 40,000 euro a year position versus a 39,999 euro a year position. Continuous segmentation is an assumption made for simplicity. We believe that accounting for discrete segmentation would not alter our main results¹.

¹Accounting for discrete segmentation would raise an important issue: how could one endogenize the different

The shape of the wage distribution is a major property of our paper that distinguishes it from other models of frictional wage dispersion. Such models belong to two main categories. First, there are papers that introduce on-the-job search. If workers search on-the-job, reservation wages are heterogenous. Under random search assumption, Burdett and Mortensen (1998) show that this heterogeneity in reservation wages implies the existence of a non-degenerate wage distribution. However, the density of the wage distribution is strictly increasing. De la Croix and Shi (forthcoming) consider a directed search version. They show that the density of the wage offer distribution is strictly decreasing. In addition, at a given initial wage, all workers prospect the same jobs, which means they all receive the same wage in the event of being hired. Second, there are papers which introduce firm heterogeneity. Van den Berg and Ridder (1998) do so while estimating the Burdett-Mortensen model. Mortensen (2000) examines productivity choices in the same model. Postel-Vinay and Robin (2002a) also endogenize firm heterogeneity, but in a model where employers can react to other firms approaching their workers by making a counteroffer (see also Postel-Vinay and Robin, 2002b). These papers manage to generate a hump-shaped wage distribution. In a close framework, but with only ex-post heterogeneity (i.e. when the quality of the match is revealed), Moscarini (2005) shows that it is possible to arrive at a wage distribution with good empirical properties (unimodal, skewed, with a Paretian right tail) with a simple Gaussian output noise. Our paper complements these studies by showing that search frictions alone can generate a non-degenerate, single-peaked and right-tailed wage distribution.

The paper thematically closest to ours are those on multiple applications. These papers modify the matching technology so that a worker can receive multiple offers at the same time. Thus, wage competition takes place at the time of choosing between different job offers. This way of analyzing the search process is very close to Stigler (1961). In Acemoglu and Shimer (2000), firms post wages and workers choose the number of offers they receive. Acemoglu and Shimer show that there is equilibrium wage dispersion. However, the density of the wage offer distribution is strictly decreasing with a mass point at its upper bound. In a similar vein, Galenianos and Kircher (2005) – GK – consider the directed search model of Albrecht, Gautier and Vroman (2005) in which firms post wages and the workers send multiple applications. Unlike Albrecht *et al.*, GK assume that firms commit to pay the posted wage irrespective of the number of job offers received by the applicant². They also obtain a strictly decreasing density of the wage distribution. In our paper, workers also send multiple applications and firms post wages. However, there are two major differences with GK. First, individuals can only receive one offer at a time in our framework. Wage competition, therefore, takes place at the time of attracting the job-seekers, and not once they have obtained several offers. Second, the intensity of search investment can vary with the wage. This is why there are only a few firms at the left and at the right of the wage distribution, while the number of jobs decreases with the wage in GK. We believe that allowing the workers to court the jobs with various degrees of aggressiveness is as important as allowing for multiple offers at a time.

wage thresholds delimiting the different market segments? Is this a problem of information (it may be necessary to save on search costs to aggregate 30,000 euro to 35,000 euro positions), or preference (30,000 euro to 35,000 euro positions offer similar standards of living, while a 40,000 euro position may change the life of the family). Interestingly, models of segmentation by job type implicitly face similar problems: what is the difference between simple jobs and complex jobs?

²Put otherwise, firms cannot react to other offers by increasing their initial wage in GK, while they can in Albrecht *et al.*

The rest of the paper proceeds as follows. Section 2 presents a model in which workers can choose their search intensity, but are bound to choose one and only one search market. We call the associated equilibrium concept the localized search equilibrium, which is basically the equilibrium of a standard directed search model with endogenous search intensity. In Section 3, workers are no longer obliged to choose a particular market. We call the associated equilibrium concept the ubiquitous search equilibrium. Section 4 studies equilibrium wage distributions. Section 5 discusses the efficiency of the decentralized outcome. Section 6 concludes.

All proofs are set forth in the appendix.

2 Localized search

What we call a localized search equilibrium is a version of Moen’s (1997) competitive search equilibrium in which the search effort is made endogenous.

2.1 Search technology

Our model follows the lines of the wage posting search model developed, among others, by Moen (1997) and Acemoglu and Shimer (1999, 2000). In this framework, firms post vacancies with non-negotiable wages. Workers, knowing all the posted wages, choose the amount of effort they will spend to search for a job. While making this choice, they are aware that if they decide to search for a job offering a wage w , they will compete with other workers seeking the same wage. Symmetrically, if a firm posts a vacancy associated with the wage w , it will compete to attract workers with the other firms offering the same wage. In other words, for each wage w , workers face a specific queue length and vacant jobs have a specific probability of being filled. For Acemoglu and Shimer (1999, 2000), such a representation of the search process recognizes that the labor market is segmented by wages and that search frictions exist within each particular sub-market (or island). Therefore, firms advertise wages, and all firms advertising a given wage and all workers applying for these jobs form a sub-market.

As in Moen (1997), we assume that an unemployed person is bound to search for a job on one and *only* one island. This is the reason why we use the term “localized” to label our equilibrium concept. Thus, the search process involves two stages: 1) firms post wages, 2) each job-seeker chooses on which sub-market she will search for a job and decides upon the amount of search effort she will spend on the previously chosen sub-market.

An unemployed person whose search effort is s bears the cost $c(s)$, but increasing effort provides more contacts with potential employers. In such a perspective, we will distinguish the amount of search effort from the *efficiency* of such an effort. When a job-seeker invests an amount of effort s , the efficiency of this effort is measured by the function $x(s)$. In the sequel, we make the following assumption on the functions $c(s)$ and $x(s)$ influencing the search process.

Assumption A1 *The cost of effort function $c : [0, +\infty) \rightarrow [0, +\infty)$ is strictly increasing, convex, twice differentiable, and satisfies $c(0) = 0$, $c'(0) = c_0 > 0$, and $c'(+\infty) = +\infty$.*

The efficiency of effort function $x : [0, +\infty) \rightarrow [0, +\infty)$ is strictly increasing, strictly concave, twice differentiable, and satisfies $x(0) = 0$, $x'(0) = +\infty$, and $x'(+\infty) = 0$.

The efficiency of effort technology is (sub)market-specific, while the cost of search depends on the overall search effort. This distinction between market-specific and overall search effort is

not useful in the localized search model, where workers must choose one and only one market before searching for a job. However, the fact that the efficiency of effort features decreasing marginal returns on each market will be crucial in the ubiquitous search model of section 3. Beyond such technical significance, Assumption A1 captures the main aspects of the job search process. Job-seeking, like the search for ideas, has two components: tiredness and efficiency. Tiredness depends on overall investment. This is why the marginal search cost depends on the aggregate search investment. Efficiency depends on the amount of resources spent on the particular market that is prospected. So far, the marginal productivity of search investment in terms of increased probability to match only depends on market-specific investment. Such marginal productivity is decreasing, which reflects the fact that courting jobs more aggressively involves tasks of increasing difficulties. For instance, it is easy to put a stamp on the letter of one's application. It is more difficult to tailor the letter to the characteristics of the job and of one's CV.³

A sub-market may be either closed – when no one enters it – or open. Which sub-markets are closed and which sub-markets are open is an outcome of the model. When there are $u(w)$ unemployed persons searching on the sub-market offering the wage w , and when the search effort of an individual on this sub-market reaches $s(w)$, the overall efficient search effort on this sub-market amounts to $\bar{x}(w)u(w)$ where $\bar{x}(w) \equiv x[s(w)]$ is the market-specific mean efficient search intensity. With $v(w)$ vacancies offering the wage w , the flow number of matches on island w is equal to $M[\bar{x}(w)u(w), v(w)]$, where the matching function satisfies the following standard assumption:

Assumption A2 *The technology $M : [0, +\infty) \times [0, +\infty) \rightarrow [0, +\infty)$ is twice continuously differentiable, strictly increasing in each of its arguments, strictly concave and linearly homogeneous. It satisfies the boundary conditions $M(U, 0) = M(0, V) = 0$, and $\lim_{U \rightarrow +\infty} M(U, V) = \lim_{V \rightarrow +\infty} M(U, V) = +\infty$.*

Let $m(\theta) \equiv M(1/\theta, 1)$. The flow probability for a vacant job offering the wage w to meet a job-seeker is:

$$m[\theta(w)] \equiv \frac{M[\bar{x}(w)u(w), v(w)]}{v(w)},$$

while the flow probability for a job seeker to meet a vacant job *per efficient unit of search* is:

$$\theta(w) m[\theta(w)] \equiv \frac{M[\bar{x}(w)u(w), v(w)]}{\bar{x}(w)u(w)}.$$

In these formulas, $\theta(w) \equiv v(w)/\bar{x}(w)u(w)$ is the market-specific tightness, that is the tightness specific to island w .

³Here, there is an analogy with the way we write our papers. Most of us write several papers at a time, and we know that they have heterogenous qualities. Why don't we only work on the best one? The reason is the following. The returns on the first hours spent on each paper is huge: your first draft was not so bad after all! Then, the hourly return of your search activity on a particular paper drops significantly (step 3 of your so intuitive result is finally proved). You may find it worthwhile to write several papers of different qualities because you can compensate a lower expected reward by a higher marginal productivity of your search investment. Finally, you will spend more time on the best paper, but you will also work on your other projects.

2.2 Agents' behavior

We study the steady state of a continuous time economy. There are a continuum of identical and infinitely-lived workers, and a continuum of firms. Each firm is associated with only one job. The measure of workers is normalized to one, while the measure of firms is endogenously determined through entry. Both are risk neutral and discount time at instantaneous rate r . Jobs can be either filled or vacant, while workers can be either employed or unemployed. A worker/firm pair produces flow output y until (exogenous) separation at rate q . Unemployed workers enjoy unemployment income z , $0 \leq z < y$, while firms endowed with a vacancy bear the flow cost h .

Following Moen (1997) and Acemoglu and Shimer (1999, 2000), we assume that workers observe all posted wages w and corresponding market tightness $\theta(w)$. Workers decide which sub-market to enter on the basis of this knowledge. Let $V_u(w)$ and $V_e(w)$ denote respectively, the value of unemployment and the value of employment on the sub-market offering the wage w . The asset value equations for $V_e(w)$ and $V_u(w)$ are given by:

$$rV_e(w) = w + q [V_u(w) - V_e(w)] \quad (1)$$

$$rV_u(w) = \max_s \{z - c(s) + x(s) \theta(w) m[\theta(w)] [V_e(w) - V_u(w)]\} \quad (2)$$

Let us denote by $R(w) \equiv rV_u(w)$ the flow gain of an unemployed person. The optimal search investment $s(w)$ responds to:

$$c'[s(w)] = x'[s(w)] \theta(w) m[\theta(w)] \frac{w - R(w)}{r + q} \quad (3)$$

$$R(w) = z - c[s(w)] + x[s(w)] \theta(w) m[\theta(w)] \frac{w - R(w)}{r + q} \quad (4)$$

The asset values of a vacancy advertised at wage w , denoted $\Pi_v(w)$, and of a filled job paying w , denoted $\Pi_e(w)$, satisfy the arbitrage equations:

$$r\Pi_v(w) = -h + m[\theta(w)] [\Pi_e(w) - \Pi_v(w)], \quad r\Pi_e(w) = y - w + q [\Pi_v(w) - \Pi_e(w)] \quad (5)$$

Consequently, when an entrepreneur decides to post the wage w , her expected gain is given by:

$$r\Pi_v(w) = \frac{-h(r + q) + m[\theta(w)] (y - w)}{r + q + m[\theta(w)]} \quad (6)$$

The main consequence of the assumption of localized search, whereby a job-seeker is constrained to search for a job on one and only one sub-market, is that competition between employers to attract workers obliges firms to offer the same expected utility for the unemployed on each open sub-market. Let us denote by V_u this common value, and by $R = rV_u = \max_w R(w)$ the associated reservation wage. This has two implications.

First, the search effort is the same on all open sub-markets. Formally, let $\Omega \in [R, y]$ be the set of potentially open sub-markets. For all $w \in \Omega$, equations (3) and (4) imply that

$$R = z - c[s(w)] + c'[s(w)] \frac{x[s(w)]}{x'[s(w)]} \quad (7)$$

Assumption A1 implies that, for a given $R \geq z$, there exists a unique search effort $s(w)$ which is the solution of equation (7). This optimal search effort does not depend on w , and we will denote it by $\sigma(R)$. It is easy to check that $\sigma'(R) > 0$.

Second, the wage on any open sub-market maximizes the utility of the unemployed. This implies that there is a monotonous relationship between tightness on open sub-markets and workers' reservation wage. Formally, for all $w \in \Omega$, equations (3) and (7) give:

$$c'[\sigma(R)] = x'[\sigma(R)] \theta(w) m[\theta(w)] \frac{w - R}{r + q} \quad (8)$$

This equality implicitly defines tightness as a function $\theta(w, R)$ of the workers' reservation wage. It is easy to check that $\theta_R(w, R) > 0$.

2.3 Localized search equilibrium

For each sub-market, firms formulate expectations concerning the associated market tightness. Given that only a subset of potential sub-markets will be opened in equilibrium, such expectations concern both equilibrium and out-of-equilibrium outcomes. We shall denote by $\tilde{\theta}(w)$ firms' common expectation on the pattern by wage of market tightness. In the sequel, we will restrain ourselves to the following hypothesis.

Assumption A3 *Let $R \geq 0$ be given. Firms' expectations are given by*

$$\tilde{\theta}(w) = \begin{cases} \theta(w, R) & \text{if } w \in [R, y] \\ 0 & \text{elsewhere} \end{cases} .$$

The idea is as follows. Firms have no reason to post a wage on a sub-market that will not be prospected by job-seekers. Yet, they must assign a value to the tightness variable when evaluating the opportunity to post a wage on a particular sub-market. They rationally expect that if this sub-market were open, it would be consistent with workers' maximization process. An entrepreneur who chooses to post the wage w expects, for a given reservation wage R , that the corresponding effort $\sigma(R)$ and market tightness $\theta(w)$ must satisfy the system of equations (7) and (8).

On the basis of this expectation $\tilde{\theta}(w)$, each entrepreneur maximizes her expected gain $\Pi_v(w)$ given by (6). Differentiating $\Pi_v(w)$ with respect to w and setting this derivative to zero yields:

$$\begin{aligned} & \left\{ m'[\tilde{\theta}(w)] \tilde{\theta}'(w) (y - w) - m[\tilde{\theta}(w)] \right\} \left\{ r + q + m[\tilde{\theta}(w)] \right\} \\ & + m'[\tilde{\theta}(w)] \tilde{\theta}'(w) \left\{ -h(r + q) + m[\tilde{\theta}(w)] (y - w) \right\} = 0 \end{aligned}$$

The free-entry condition, $\Pi_v(w) = 0$, implies that the last term between brackets vanishes and the optimal market wage is characterized by the following equation:

$$\alpha(\tilde{\theta}(w)) \frac{\tilde{\theta}'(w)}{\tilde{\theta}(w)} = -\frac{1}{y - w} \quad (9)$$

where $\alpha(\theta) = -\theta m'(\theta) / m(\theta) \in (0, 1)$ is the elasticity of the recruitment rate with respect to market tightness.

On the other hand, differentiating relation (8) with respect to w , one obtains:

$$\frac{\tilde{\theta}'(w)}{\tilde{\theta}(w)} = \frac{\theta_w(w, R)}{\theta(w, R)} = -\frac{1}{[1 - \alpha(\theta(w, R))](w - R)}$$

Substituting this expression of $\tilde{\theta}'(w)/\tilde{\theta}(w)$ into (9) gives the optimal market wage as a function of reservation wage R and market tightness $\tilde{\theta}(w)$:

$$w = \alpha(\tilde{\theta}(w))y + [1 - \alpha(\tilde{\theta}(w))]R \quad (10)$$

In equilibrium, equation (6) implies that the market tightness $\theta(w)$ must satisfy:

$$m[\theta(w)] = \frac{h(r+q)}{y-w} \quad (11)$$

The consistency of expectations implies that $\tilde{\theta}(w) = \theta(w)$ in equilibrium. This yields

$$w = \alpha(\theta(w))y + [1 - \alpha(\theta(w))]R \quad (12)$$

The following Lemma shows that this equation defines a unique wage for a given R . This wage is the wage that would be determined in a Nash bargain with the worker bargaining power parameter equal to the Hosios value, that is the elasticity of the matching technology with respect to unemployment.

Lemma 1 *Let $\phi : [0, y] \times \mathbb{R} \rightarrow \mathbb{R}$ be such that*

$$\phi(w, R) \equiv \alpha(\theta(w))y + [1 - \alpha(\theta(w))]R - w \quad \text{with} \quad \theta(w) = m^{-1}\left[\frac{h(r+q)}{y-w}\right],$$

For all $R \in [0, y]$, $\phi(w, R) = 0$ has a unique root in w .

The properties of the localized search equilibrium are summarized in the following proposition.

Proposition 1 LOCALIZED SEARCH EQUILIBRIUM

Under Assumptions A1, A2 and A3,

(i) A localized search equilibrium is characterized by a quadruplet $(\theta^, s^*, w^*, R^*)$ corresponding to the equilibrium value of the labor market tightness, the search effort, the wage and the flow gain of an unemployed person that satisfies:*

$$m(\theta) = \frac{h(r+q)}{y-w} \quad (13)$$

$$c'(s) = x'(s)\theta m(\theta) \frac{w-R}{r+q} \quad (14)$$

$$R = z - c(s) + x(s)\theta m(\theta) \frac{w-R}{r+q} \quad (15)$$

$$w = \alpha(\theta)y + [1 - \alpha(\theta)]R \quad (16)$$

(ii) There exists a unique localized search equilibrium.

Using equations (13) to (16), it is possible to show that s^* and θ^* are the solutions of the following system:

$$\frac{h}{m(\theta^*)} = \frac{[1 - \alpha(\theta^*)][y - z + c(s^*)]}{r + q + x(s^*)\alpha(\theta^*)\theta^*m(\theta^*)}, \quad c'(s^*) = \frac{\alpha(\theta^*)}{1 - \alpha(\theta^*)}h\theta^*x'(s^*) \quad (17)$$

These equations will prove useful while studying the efficiency properties of the localized search equilibrium (see Proposition 7 in Section 5).

To complete the characterization of the localized search equilibrium, it remains to define the unemployment rate. As the equilibrium wage is unique, there is a unique opened (sub-)market. On this market, the job finding rate, denoted by λ^* , is given by:

$$\lambda^* = x[\sigma(R^*)]\theta^*m(\theta^*),$$

and the stationary equilibrium unemployment rate, denoted by u^* , stems from the equality between the flows in and out of employment, i.e. $q(1 - u^*) = \lambda^*u^*$. Finally, one has:

$$u^* = \frac{q}{q + \lambda^*}$$

For our purpose, the main result is that there is a unique equilibrium wage when job search is localized. We now consider the labor market equilibrium when we relax the assumption of localized search.

3 Ubiquitous search

3.1 Agents' behavior with ubiquity

As in the previous section, the search market is segmented by wage. The behavior of entrepreneurs remains unchanged, i.e. firms post vacancies with associated non-negotiable wages. However, we now assume that an unemployed person is able to search simultaneously on *every* existing sub-market. Hence, there is ubiquity on the search market: a job-seeker is not bound to search on a single sub-market. Ubiquity means that the worker has to decide on the search investment on *every* existing sub-market.

If there are u unemployed persons in the economy, the overall search effort on the sub-market offering the wage w now amounts to $\bar{x}(w)u$ where $\bar{x}(w)$ still denotes the market-specific mean efficiency of search efforts. With $v(w)$ vacancies offering the wage w , the flow number of matches on sub-market w is equal to $M[\bar{x}(w)u, v(w)]$, where the matching function still satisfies Assumption A2. Consequently, the flow probability for a vacant job to meet a job-seeker and the flow probability for a job-seeker to meet a vacant job per efficient unit of search on the sub-market offering the wage w are given by:

$$m[\theta(w)] \equiv \frac{M[\bar{x}(w)u, v(w)]}{v(w)}, \quad \theta(w)m[\theta(w)] \equiv \frac{M[\bar{x}(w)u, v(w)]}{\bar{x}(w)u},$$

where market-specific tightness is now defined by $\theta(w) \equiv v(w)/\bar{x}(w)u$.

Let us denote by V_u the expected lifetime utility of an unemployed individual. If this person takes a job paying w , she obtains the lifetime utility $V_e(w)$ described by the arbitrage equation:

$$rV_e(w) = w + q[V_u - V_e(w)]$$

It follows that the reservation wage, R , is always such that $R = rV_u$. A priori, the set of possible wages – equivalently, the set of islands – covers the entire interval $[z, y]$, but the set of islands that will be visited belongs to the interval $[R, y]$. As in the previous section, each job-seeker observes the posted wage w and the corresponding labor market tightness $\theta(w)$ on each sub-market.

However, an unemployed person has to choose the set $\{s(w)\}$ of search efforts that she will *simultaneously* exert on *each* sub-market w . Let us denote by $S = \int_R^y s(w) dw$ the total search effort, the expected gain of a job seeker reads:

$$rV_u = \max_{s(\cdot)} \left\{ z - c(S) + \int_R^y x[s(w)] \theta(w) m[\theta(w)] [V_e(w) - V_u] dw \right\} \quad (18)$$

It must be stressed that unlike other models with multiple applications (see for instance GK, or Albrecht *et al.*, 2006), workers can only receive one offer at a time in our framework. This property is highly convenient, since it makes the working of our model very similar to the standard job search model. In particular, workers follow a reservation wage strategy while deciding which market to prospect. The reason why there are no multiple offers can be understood as follows. Suppose that the search space is actually composed of a discrete number of matching places, each of size dw . Hence, there are $(y - R) / dw$ sub-markets. The probability of receiving an offer from the subset $[w, w + dw]$ during the interval of time dt is $x[s(w)] \theta(w) m[\theta(w)] dw dt$. This probability decreases with the number of matching places. As dw tends to 0, the number of matching places tends to infinity, and the probability of receiving an offer on a particular sub-market tends to 0. As a result, the probability of receiving more than one offer at a time tends to 0 as the interval of time dt also tends to 0. We give a complete proof of the result at the end of the Appendix.

On sub-market w , the optimal search effort of an unemployed person is characterized by the first-order condition:

$$c'(S) = x'[s(w)] \theta(w) m(\theta(w)) \frac{w - R}{r + q}, \quad \text{for all } w \in [R, y] \quad (19)$$

And the equation (18) defining V_u becomes:

$$R = z - c(S) + \int_R^y x[s(w)] \theta(w) m(\theta(w)) \frac{w - R}{r + q} dw \quad (20)$$

The first-order condition (19) states that, in every sub-market, the marginal cost of searching for a job must be equal to the marginal gain of this activity. This relation highlights the fact that the search technology $x(s)$ must exhibit marginal decreasing returns to obtain a definite search effort associated with each sub-market. Indeed, under constant marginal returns to search, a job-seeker would allocate their entire search investment to the sub-market that yields the largest reward. Such a sub-market offers the best combination of wage and employment probability, i.e. it gives the greatest expected utility gain represented by the product $\theta(w) m(\theta(w)) (w - R)$. Hence, despite the fact that workers may be allowed to search on several sub-markets at a time, they would not use this possibility and only one market would be opened. This result no longer applies with marginal decreasing returns in the search technology. A worker can then compensate a lower reward by investing less, which raises the marginal productivity of the search effort and leaves the marginal benefit to search unchanged.

Consequently, there is no longer a unique value of the search effort: the search investment varies on each prospected sub-market. Moreover, equations (19) and (20) imply that the reservation wage R and the collection of search efforts $\{s(w)\}$ are linked by

$$R = z - c(S) + c'(S) \int_R^y \frac{x[s(w)]}{x'[s(w)]} dw \quad (21)$$

$$S = \int_R^y s(w) dw \quad (22)$$

Tightness on each sub-market must satisfy equation (19). This equation defines tightness as a function $\theta(w, R, \{s(w)\})$ in which the reservation wage R and the set of search efforts $\{s(w)\}$ are linked by (21) and (22).

3.2 Ubiquitous search equilibrium

As in the localized search case, for each sub-market, firms formulate expectations concerning the associated tightness. We shall still denote by $\tilde{\theta}(w)$ firms' common expectation on the pattern by wage of market tightness. We consider the following assumption.

Assumption A3' *Let $R \geq z$ and $\{s(w)\}_{w \geq R}$ such that (21) and (22) hold. Firms' expectations are given by*

$$\tilde{\theta}(w) = \begin{cases} \theta(w, R, \{s(w)\}) & \text{if } w \in [R, y] \\ 0 & \text{elsewhere} \end{cases} .$$

Assumption A3' is a mere adaptation of Assumption A3 in the context of ubiquitous search. First, firms have no reason to post a wage on a sub-market that will not be prospected by the job-seekers. Second, they must assign a value to the tightness variable when evaluating the opportunity to post a wage on a particular sub-market. They rationally expect that if this sub-market were opened, it would be consistent with workers' maximization process. Consequently, when an entrepreneur chooses to post a wage equal to w , she considers that, for a given reservation wage R and a given collection $\{s(w)\}_{w \geq R}$ of search efforts satisfying (21) and (22), the market tightness must satisfy the equation (19).

The asset values $\Pi_v(w)$ and $\Pi_e(w)$ of a vacancy posting a wage w and of a filled job paying this wage are still defined by the relations (5). On the basis of her expectation $\tilde{\theta}(w)$ defined in Assumption A3', each entrepreneur can maximize her expected gain $\Pi_v(w)$ given by (6). Formally, the entrepreneur's problem is the same as in the case with localized search. Thus, when the free-entry condition $\Pi_v(w) = 0$ is satisfied, the equilibrium value of the market tightness function $\theta(w)$ is still given by equation (11) for any wage in the interval $[R, y]$. It is worth noting that the latter equation means that a firm advertising a high wage vacancy expects this vacancy to be filled quickly – $m(\theta(w))$ has to be large – while a firm advertising a low wage vacancy can wait longer. This explains why the equilibrium value of the tightness function must be decreasing with the wage.

Proposition 2 UBIQUITOUS SEARCH EQUILIBRIUM

Under Assumptions A1, A2 and A3',

(i) A ubiquitous search equilibrium is characterized by a quadruplet $(\theta^*(w), s^*(w), S^*, R^*)$ corresponding to the equilibrium value of the labor market tightness function, the search effort function, the global effort and the flow gain of an unemployed person, that satisfies:

$$m[\theta(w)] = \frac{h(r+q)}{y-w}, \forall w \in [R, y] \quad (23)$$

$$c'(S) = x'[s(w)] \theta(w) m[\theta(w)] \frac{w-R}{r+q}, \forall w \in [R, y] \quad (24)$$

$$R = z - c(S) + \int_R^y x[s(w)] \theta(w) m[\theta(w)] \frac{w-R}{r+q} dw \quad (25)$$

$$S = \int_R^y s(w) dw \quad (26)$$

(ii) There exists a unique ubiquitous search equilibrium.

The ubiquitous search equilibrium displays two main features. First, it is unique. Second, all wages belonging to the interval $[R, y]$ are prospected, so that $[R, y]$ is also the support of the equilibrium wage distribution. These two features are deeply related to each other: they crucially depend on the ubiquity assumption – the fact that there are no restrictions on the size of prospected markets. Indeed, workers follow a reservation wage strategy. Hence, they prospect all the markets that offer a wage larger than the reservation wage R . Without loss of generality, suppose for instance there is an interval $[a, b] \subset [R, y]$ such that wages belonging to $[a, b]$ are not offered in equilibrium. Consider the wage \hat{w} such that $a < \hat{w} \leq b$. An employer offering the wage b obtains $(y-b)/(r+q)$ if the job is filled, or obtains 0 if it remains vacant. This employer may decide to offer the wage \hat{w} . In such a case, he would be the only employer on the corresponding sub-market, and, therefore, the probability of matching would be one in any time interval. His profit would be $(y-\hat{w})/(r+q) > (y-a)/(r+q)$. Of course, such a profit opportunity would be exploited, and employers would enter this new market until the free entry condition is satisfied. Stated otherwise, there are no holes in the equilibrium set of offered wages. Of course, the key reason relies on the fact that workers cannot be captured on a particular subset of the search space.

To end characterization of the ubiquitous search equilibrium, the unemployment rate remains to be defined. The job-finding rate is worth

$$\lambda = \int_R^y x[s(w)] \theta(w) m[\theta(w)] dw,$$

that is the sum of the different rates of contact over the different markets that the job-seekers prospect. Unemployment can then be computed from the equality between flows in and out of unemployment: $q(1-u) = \lambda u$. The unemployment rate is worth $u = q/(q+\lambda)$.

We now turn to the properties of the equilibrium.

3.3 Market-specific search investment

Differentiating the logarithm of both sides of equation (24) with respect to w , we get:

$$-\frac{x''[s(w)]}{x'[s(w)]}s'(w) = \frac{\theta'(w)}{\theta(w)}[1 - \alpha(\theta(w))] + \frac{1}{w - R} \quad (27)$$

while differentiating (23) with respect to w still gives (9). Then, eliminating $\theta'(w)/\theta(w)$ between (27) and (9) one obtains:

$$s'(w) \equiv -\frac{x'[s(w)]}{x''[s(w)]} \frac{\alpha(\theta(w))y + [1 - \alpha(\theta(w))]R - w}{\alpha(\theta(w))(y - w)(w - R)} \quad (28)$$

Equation (27) shows that the wage has two conflicting effects on search effort. On the one hand, there is a positive direct effect. At given market tightness, a higher wage raises the return to search, thereby motivating search investment (this effect is captured by the positive term $1/(w - R)$). On the other hand, there is a negative indirect effect. Indeed, market tightness decreases with the wage. A higher wage deteriorates the search prospects, thereby reducing search investment (this effect is captured by the negative term $[1 - \alpha(\theta(w))]\theta'(w)/\theta(w)$). Therefore the sign of $s'(w)$ seems ambiguous. We can go further by noticing that the function $\phi(w, R)$ defined in Lemma 1 appears at the right-hand side of equation (28). More precisely, one can see that $s'(w)$ has the same sign as $\phi(w, R)$. This remark enables us to state the following proposition.

Proposition 3 PATTERN OF SEARCH INVESTMENT

Under Assumptions A1 to A3, the effort function $s : [R, y] \rightarrow [0, +\infty)$ is \cap -shaped and satisfies $s(R) = s(y) = 0$

The key finding of Proposition 3 is the non-monotonicity of the relationship between wage and search investment depicted by Figure 1. The pattern of search investment reflects the pattern of marginal reward to search. It should be recalled that such reward involves a peculiar combination of wage and employment probability given by the expected utility gain $\theta(w)m[\theta(w)](w - R)$. Search investment is thus very small at both low and high wages. In the former case, employment probability represented by $\theta(w)m[\theta(w)]$ is large, but it is not worth investing much as the wage w is close to the reservation wage R . In the latter case, the wage may be very good, but job opportunities collapse. More generally, the direct positive effect of the wage on search investment dominates at low wages, while the indirect negative effect due to lower tightness dominates at higher wages. The search investment then reaches a maximum on the market where the reward is the highest.

Let us denote by $w_1(R)$ the root of equation $\phi(w, R) = 0$ that gives the largest search investment (see Figure 1). This wage is defined by $w_1(R) = \alpha[\theta(w_1(R))]y + \{1 - \alpha[\theta(w_1(R))]\}R$. For R given, this wage is the only wage offer in the localized search equilibrium (see equation (12)).

4 Wage distributions with ubiquity

The purpose of this section is to analyse the shape of the wage distribution that is implied by our model. We proceed in three steps. First, we focus on the equilibrium wage offer distribution.

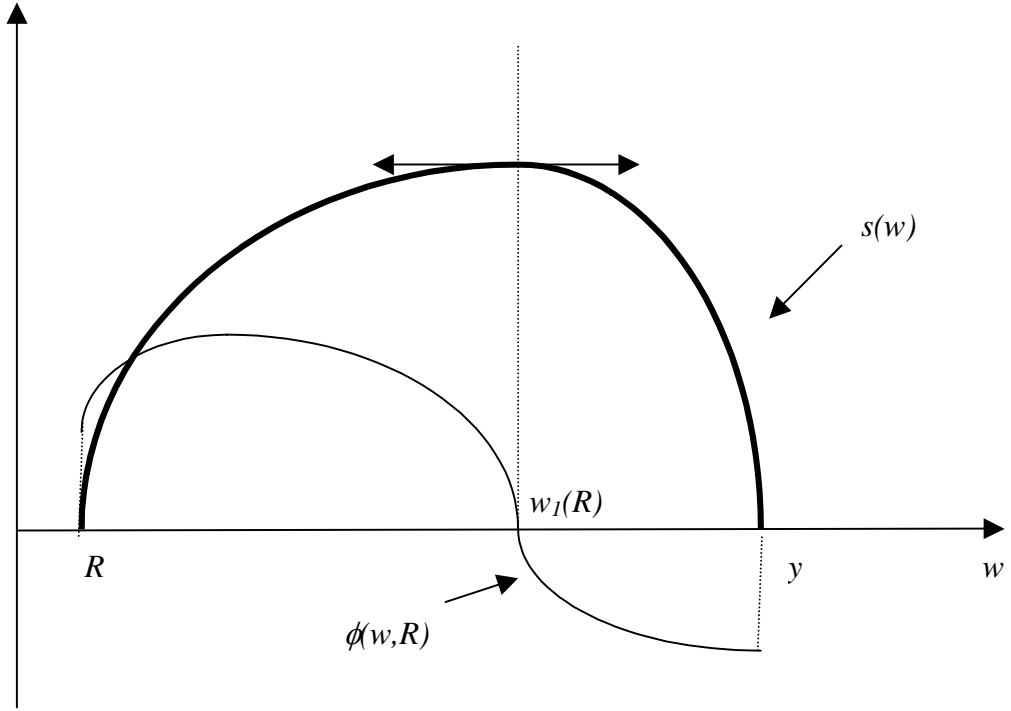


Figure 1: Search investment by wage

Second, we analyse the wage distribution among employed workers. Third, we consider an example.

4.1 Wage offer distribution

The number of vacancies advertised at wage w is worth $v(w) = \theta(w) x [s(w)] u$, where u represents the unemployment rate. The total number of vacancies is thus $v = \int_R^y v(w) dw$. The cdf and the pdf of the wage offer distribution are then defined by

$$F(w) = \frac{\int_R^w v(\xi) d\xi}{v}, \quad F'(w) = \frac{v(w)}{v} = \theta(w) x [s(w)] \frac{u}{v} \quad (29)$$

How does the density change with the wage? Taking the second derivative of F yields

$$\frac{F''(w)}{F'(w)} \equiv \frac{\theta'(w)}{\theta(w)} + \frac{x'[s(w)] s'(w)}{x [s(w)]} \quad (30)$$

Changes in the density of the wage offer distribution reflect changes in the number of vacancies associated with each wage (i.e. $v'(w)/v(w) = F''(w)/F'(w)$). Equation (30) shows that such changes result from two main factors: the pattern of market tightness by wage on the one hand, and the pattern of search investment by wage on the other hand. Hence, the right-hand side of equation (30) is composed of two terms. The first term is negative and reflects the fact that tightness is strictly decreasing in wage. Due to this term, the density of the wage offer distribution tends to decrease with the wage as the number of job offers per unit of search declines as the wage raises. The second term depicts the influence of search investments. It is

non-monotonous, reflecting the non-monotonicity of $s(w)$. More precisely, it is positive at wages close to the lower bound of the support $[R, y]$ of the wage offer distribution, while it becomes negative at wages close to the upper bound.

With the help of (9) and (28), we have:

$$\frac{F''(w)}{F'(w)} = \frac{\gamma(w) \{ \alpha(\theta(w)) y + [1 - \alpha(\theta(w))] R - w \} - (w - R)}{\alpha(\theta(w)) (y - w)(w - R)} \quad (31)$$

with $\gamma(w) = -\frac{x'^2[s(w)]}{x[s(w)]x''[s(w)]} > 0$. It appears that $F''(w)$ has the same sign as the function $\psi(w, R) \equiv \gamma(w)\phi(w, R) - (w - R)$. Hence, the properties of the wage offer distribution will depend on the number of roots of the equation $\psi(w, R) = 0$. The following assumption will be useful to obtain more precise results.

Assumption A4 Let $\psi : [0, y] \times \mathbb{R} \rightarrow \mathbb{R}$ be such that

$$\psi(w, R) = \gamma(w)\phi(w, R) - (w - R)$$

For all $R \in [0, y]$, $\psi(w, R) = 0$ has a unique root in w .

One can check that this assumption is satisfied with a Cobb-Douglas matching function (α is then a constant) and with an isoelastic efficiency search function (γ is then a constant).

Proposition 4 PROPERTIES OF THE WAGE OFFER DISTRIBUTION

Under Assumptions A1 to A3,

(i) *Non-monotonicity.* The wage offer distribution $F : [R, y] \rightarrow [0, 1]$ is non-monotonous and satisfies $F'(R) = F'(y) = 0$.

(ii) *Single peak.* If in addition A4 holds, the wage offer distribution is \cap -shaped

(iii) *Right tail.* If $\lim_{\theta \rightarrow 0} \alpha(\theta) > 0$ and $\lim_{w \rightarrow y} \gamma(w) < \infty$, $F''(y) = 0$

We obtain three results. First, the density of the wage offer distribution is non-monotonous. This is in sharp contrast with the literature discussed in the introduction, which predicts either increasing or decreasing density of the wage offer distribution. In fact, the result is induced by the non-monotonicity of the pattern of search investment by wage level. If search investment were unable to vary with the wage, the density of the wage offer distribution would be strictly decreasing, only reflecting the decreasing pattern of tightness with respect to the wage. Note that $\psi(w_1(R), R) < 0$: the peak of the wage offer distribution corresponds to a lower wage than the peak of the search investment function. This reflects the fact that tightness is strictly decreasing in wage. Second, the density is single-peaked provided some additional (yet not too demanding) restrictions on the matching technology and the efficiency of effort function hold. Third, the wage offer distribution generally has a flat tail at its upper bound.

4.2 Actual wage distribution

As search intensity varies with the wage level, the *actual* wage distribution (i.e. the distribution of wages among employees, which coincides with the wage distribution among newly employed workers) departs from the wage offer distribution. Let $G(w)$ be the cdf of the actual wage distribution among employees. This can be deduced from a standard flow equilibrium reasoning.

For each wage $w \in [R, y]$, the outflow from the pool of those employed who earn less than w equals the inflow from the pool of unemployed:

$$q(1-u)G(w) = u \int_R^w x[s(\xi)]\theta(\xi)m[\theta(\xi)]d\xi$$

Since $q(1-u) = \lambda u$, and remembering that $v(w) = vF'(w) = x[s(w)]\theta(w)u$, it ensures that:

$$G(w) = \frac{v}{\lambda u} \int_R^w F'(\xi)m[\theta(\xi)]d\xi$$

Thus, one has:

$$G'(w) = \frac{v}{\lambda u} F'(w)m[\theta(w)] \quad (32)$$

Differentiating this latter equality with respect to w and taking into account (9) gives:

$$\frac{G''(w)}{G'(w)} = \frac{F''(w)}{F'(w)} - \alpha(\theta(w)) \frac{\theta'(w)}{\theta(w)} = \frac{F''(w)}{F'(w)} + \frac{1}{y-w} \quad (33)$$

Using relation (31) that defines $F''(w)$, one obtains:

$$\frac{G''(w)}{G'(w)} = \frac{\psi(w, R) + \alpha(\theta(w))(w-R)}{\alpha(\theta(w))(y-w)(w-R)} \quad (34)$$

It appears that $G''(w)$ has the same sign as the function $\chi(w, R) \equiv \psi(w, R) + \alpha(\theta(w))(w-R)$. Hence, the properties of the actual wage distribution depend on the number of roots of the equation $\chi(w, R) = 0$. The following assumption will be useful to obtain more precise results.

Assumption A5 Let $\psi : [0, y] \times \mathbb{R} \rightarrow \mathbb{R}$ be such that

$$\chi(w, R) = \psi(w, R) + \alpha(\theta(w))(w-R)$$

For all $R \in [0, y]$, $\chi(w, R) = 0$ has a unique root in w .

One can check that this assumption is satisfied with a Cobb-Douglas matching function (α is then a constant) and with an isoelastic efficiency search function (γ is then a constant).

Proposition 5 PROPERTIES OF THE ACTUAL WAGE DISTRIBUTION

Under Assumptions A1 to A3,

(i) *Non-monotonicity.* The actual wage distribution $G : [R, y] \rightarrow [0, 1]$ is non-monotonous and satisfies $G'(R) = G'(y) = 0$.

(ii) *Stochastic dominance.* $G(w) < F(w)$ for all $w \in (R, y)$

(iii) *Single-peak.* If in addition A5 holds, the actual wage distribution is \cap -shaped

As with the wage offer distribution, the actual wage distribution features properties that are remarkably consistent with the facts: non-monotonous and generally single-peaked. However, unlike the wage offer distribution, the actual wage distribution is not always right-tailed. Note that the wage offer distribution first-order stochastically dominates the actual wage distribution. It means that individuals confronted with both distributions would unambiguously choose the latter. Such a result is hardly surprising: job-seekers observe the wage offer distribution and alter the wage they will be paid later by modulating their search investment on each sub-market. This optimization process makes the actual wage distribution looks better than the wage offer distribution.

4.3 A Cobb-Douglas example

We end this section by considering usual explicit forms for the matching function and the efficiency of effort function. In the sequel, we will refer to this particular case as the Cobb-Douglas example. It appears that with such specifications, the wage offer distribution and the actual wage distribution are strongly linked with a well-known statistical distribution, the Beta distribution.

Proposition 6 THE COBB-DOUGLAS EXAMPLE

Assume that $m(\theta) = M_0\theta^{-\alpha}$, $M_0 > 0$, $\alpha \in (0, 1)$ and $x(s) = s^{\frac{\gamma}{1+\gamma}}$, $\gamma > 0$. Let $\omega = (w - R) / (y - R)$ be the normalized wage, and also let H_F be the cdf of the normalized wage offer distribution, while H_G is the cdf of the actual normalized wage distribution. Then,

(i) H'_F is the density of a $\beta\left(\frac{1-\alpha}{\alpha}\gamma + \frac{1}{\alpha} + 1, \gamma + 1\right)$ distribution, that is

$$H'_F(\omega) = \frac{(1-\omega)^{\frac{1-\alpha}{\alpha}\gamma + \frac{1}{\alpha}} \omega^\gamma}{B\left(\frac{1-\alpha}{\alpha}\gamma + \frac{1}{\alpha} + 1, \gamma + 1\right)}, \forall \omega \in [0, 1]$$

(ii) H'_G is the density of a $\beta\left(\frac{1-\alpha}{\alpha}(\gamma + 1) + 1, \gamma + 1\right)$ distribution, that is

$$H'_G(\omega) = \frac{(1-\omega)^{\frac{1-\alpha}{\alpha}(\gamma + 1)} \omega^\gamma}{B\left(\frac{1-\alpha}{\alpha}(\gamma + 1) + 1, \gamma + 1\right)}, \forall \omega \in [0, 1]$$

where B is the Beta function such that

$$B(t_1 + 1, t_2 + 1) = \int_0^1 (1-\xi)^{t_1} \xi^{t_2} d\xi$$

The Cobb-Douglas example displays several appealing features. First, we can find a normalization of the wage such that the offer distribution and actual distribution of such normalized wage follow simple Beta distributions. Second, the parameters of the Beta distributions only involve elasticity of the matching function and elasticity of effort function. We do not need to solve the model to find the shape of the different wage distributions. Third, wage distributions are both single-peaked. Fourth, the wage offer distribution has a flat right tail. Moreover,

$$F''(R) = \gamma \frac{(y-R)^{\frac{1-\alpha}{\alpha}\gamma + \frac{1}{\alpha}}}{\int_R^y (y-\xi)^{\frac{1-\alpha}{\alpha}\gamma + \frac{1}{\alpha}} (\xi-R)^\gamma d\xi} \lim_{w \rightarrow R} (w-R)^{\gamma-1}$$

Thus $F''(R)$ can either be nil or infinite depending on whether γ is larger or lower than one. Fifth, we can highlight the parameter circumstances under which the actual wage distribution has a flat right tail. Indeed, $G''(y) = 0$ if $\gamma > \frac{2\alpha-1}{1-\alpha}$ and $G''(y) = -\infty$ if $\gamma < \frac{2\alpha-1}{1-\alpha}$. Thus, the actual wage distribution is right-skewed when the parameters of the matching function and the search function satisfy $\gamma > \frac{2\alpha-1}{1-\alpha}$. Right-skewness is not a systematic property but can occur for a wide range of parameters of the model. Similarly, we can show that $G''(R) = 0$ if $\gamma > 1$ and $G''(R) = \infty$ if $\gamma < 1$. Therefore, the Cobb-Douglas case is consistent with an actual wage distribution characterized by a single peak, a flat right tail, and no left tail. This is so when $\frac{2\alpha-1}{1-\alpha} < \gamma < 1$. The following figure depicts the pdf of the wage offer and actual wage distributions in such a case.

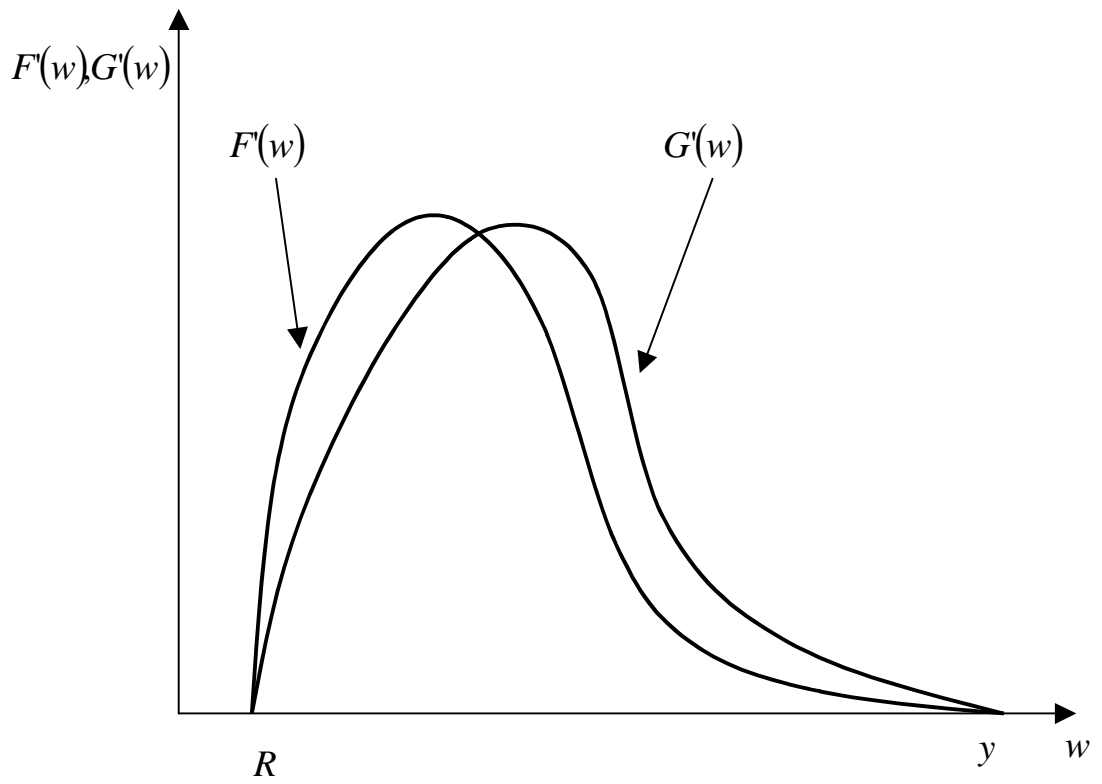


Figure 2: Wage offer and actual wage distributions - case $\frac{2\alpha-1}{1-\alpha} < \gamma < 1$

Finally, note the role played by the parameter γ of the efficiency of effort function. When γ tends to 0, the search intensity is the same in each sub-market. The shape of the wage distribution only reflects the pattern of market tightness by wage. The density of the wage offer distribution as well as the density of the actual wage distribution are then strictly decreasing in wage. Conversely, when γ tends to infinity, the efficiency of effort function has constant marginal returns. As a result, workers concentrate their search investment in the sub-market where the returns are the highest. Both the wage offer distribution and the actual wage distribution collapse to a single wage, the only wage offer of the localized search equilibrium.

From an empirical perspective, the Beta distribution should be rejected by the data because it does not feature the Paretian tail typical of empirical wage distributions. Yet, two points should be made. On the one hand, such Beta distribution is obtained for homogeneous firms and workers. Introducing some heterogeneity on the firm/worker side should make the Cobb-Douglas example compatible with a Paretian-tailed aggregate wage distribution. On the other hand, the main objective of the Cobb-Douglas example is to illustrate our main results. Another parameterization of the effort function can generate wage distributions that fit the empirical wage distributions better. But, of course, at the cost of losing the simplicity of the Cobb-Douglas example.

5 Efficiency

In this section, we compare the decentralized outcome to the efficient allocation. This comparison is made under the two cases highlighted so far, i.e. when search is localized and when workers are ubiquitous. We show that the localized search equilibrium is efficient, while the ubiquitous search equilibrium is not. We proceed in two steps. First, we compute efficient allocations at a given number of matching places. Second, we endogenize the number of matching places.

5.1 Efficient allocations at a given number of matching places

The main conceptual difficulty associated with efficient allocation relies on the segmentation of the search place. In a decentralized economy, the search market is segmented by wage: each wage is associated to an autonomous sub-market. It means that market segmentation requires wage dispersion. For the planner's problem, we shall assume that the search place is segmented: in this sub-section, we assume as Moen (1997) that the mass-number of matching places (islands, for short) is given.

Let I be the measure of islands, and $i \in [0, I]$ be their index. Under localized search, the unemployed are bound to search for a job on a single matching place. The benevolent planner chooses the number of unemployed $u(i)$ and the number of vacancies $v(i)$ assigned to island i . The overall unemployment rate is then given by $u = \int_0^I u(i) di$. Under ubiquitous search there are no restrictions on the number of prospected places, and $u(i) = u$ in each island. In both cases, the planner sets the search effort $s(i)$ of workers seeking a job on island i . As a consequence, the total cost of search investment is defined by $\int_0^I c(s(i)) u(i) di$ when search is localized and by $uc \left(\int_0^I s(i) di \right)$ when search is ubiquitous. The tightness specific to island i is given by $\theta(i) = v(i) / [x(s(i)) u]$ in case of ubiquitous search and by $\theta(i) = v(i) / [x(s(i)) u(i)]$ in case of localized search. In both cases, the job-finding rate specific to island i is equal to

$x(s(i))\theta(i)m[\theta(i)]$. When search is localized, we have:

$$\dot{u}(i) = q[1 - u(i)] - u(i)x(s(i))\theta(i)m(\theta(i)) \quad \text{for all } i \in [0, I] \quad (35)$$

While when search is ubiquitous, the dynamic of unemployment is:

$$\dot{u} = q(1 - u) - u \int_0^I x(s(i))\theta(i)m(\theta(i)) \, di \quad (36)$$

The instantaneous net social products in case of localized search and in case of ubiquitous search are respectively given by:

$$\omega = y \left(1 - \int_0^I u(i) \, di \right) + z \int_0^I u(i) \, di - c \left(\int_0^I s(i) u(i) \, di \right) - h \int_0^I \theta(i)x(s(i))u(i) \, di$$

$$\omega = y(1 - u) + uz - uc \left(\int_0^I s(i) \, di \right) - hu \int_0^I \theta(i)x(s(i)) \, di$$

The planner's problem is to maximize the discounted social product $\int_0^{+\infty} \omega e^{-rt} \, dt$ with respect to the relevant variables $s(i), \theta(i), u(i)$ or u , and subject to the relevant law of motion, that is (36) or (35).

The following result describes the stationary solutions of this maximization program for each search environment.

Proposition 7 EFFICIENT ALLOCATIONS

Under Assumptions A1 and A2, for any given number I of search places

(i) *In the localized search case, there is a unique stationary efficient allocation such that $s(i) = s^l$ and $\theta(i) = \theta^l$ for all $i \in [0, I]$, with*

$$\frac{h}{m(\theta^l)} = \frac{[1 - \alpha(\theta^l)] [y - z + c(s^l)]}{r + q + x(s^l)\alpha(\theta^l)\theta^l m(\theta^l)} \quad (37)$$

$$c'(s^l) = \frac{\alpha(\theta^l)}{1 - \alpha(\theta^l)} h\theta^l x'(s^l) \quad (38)$$

(ii) *In the ubiquitous search case, there is a unique stationary efficient allocation such that $s(i) = s^u$ and $\theta(i) = \theta^u$, for all $i \in [0, I]$, with*

$$\frac{h}{m(\theta^u)} = \frac{[1 - \alpha(\theta^u)] [y - z + c(Is^u)]}{r + q + x(s^u)\alpha(\theta^u)\theta^u m(\theta^u) I} \quad (39)$$

$$c'(Is^u) = \frac{\alpha(\theta^u)}{1 - \alpha(\theta^u)} h\theta^u x'(s^u) \quad (40)$$

Comparing (37) and (38) with their decentralized counterpart (17) shows that the localized search equilibrium is efficient. This result is very similar to Moen (1997). Wage-posting can thus decentralize the efficient allocation.

When search is ubiquitous, part (ii) of the proposition shows that the planner sets the same tightness and the same search intensity for all individuals in each island of the interval $[0, I]$. Tightness and search intensity are decreasing in the measure I of islands. It follows that the ubiquitous search equilibrium is inefficient. Consider for instance the case where $I = y - R$, with R the equilibrium reservation wage. In this case, the number of opened matching places is the same in the social optimum and in the decentralized economy. However, search investment varies from one sub-market to another in the decentralized economy, while it is constant in each island at the social optimum.

The reason for inefficiency is very close to GK. When workers make multiple job applications (or search simultaneously on several sub-markets), the expected number of matches is maximized by allocating applications uniformly across jobs (by spreading search effort uniformly across sub-markets). Julien, Kennes and King (2006) also give a simple argument along these lines. However, the decentralized allocation features wage dispersion. It follows that search investment varies across jobs, violating efficiency.

5.2 On the number of matching places

In this sub-section, we discuss the optimal number of matching places that would be chosen by the social planner.

Let us begin with the localized search case. Proposition 7 shows that at a given number of matching places, all allocations featuring a search intensity s^l and a tightness θ^l are efficient. Owing to constant returns to scale in the matching technology and due to the fact that each worker must be assigned to a single search place, the stationary social product does not depend on the number of matching places. As a result, the efficient number of matching places is indeterminate under localized search. If there were a fixed cost associated with each search place, the planner would only create a single matching place.

Now, we turn to the ubiquitous search environment. To simplify, consider the case where the discount rate r tends to 0. Then, efficient allocation maximizes the stationary social product. The optimal number of market places results from:

$$\max_{I \geq 0} \{ \omega(I) = (1 - u(I))y + u(I)[z - c(Is^u(I))] - hu(I)I\theta^u(I)x^u(I) \} \quad (41)$$

where θ^u and s^u are defined by Proposition 7, and $u(I) = q/[q + \theta^u(I)m(\theta^u(I))x(s^u(I))I]$. The derivative of the objective with respect to I is:

$$\omega'(I) = -\frac{\partial u(I)}{\partial I} [y - z + c(Is^u(I)) + hI\theta^u(I)x^u(I)] - u(I)s^u(I)c'(Is^u(I)) - hu(I)\theta^u(I)x(s^u(I)) \quad (42)$$

Using equations (39) and (40), we obtain:

$$\omega'(I) = \frac{\alpha}{1 - \alpha} hu(I)\theta^u(I)x(s^u(I)) [1 - s^u(I)x'(s^u(I))/x(s^u(I))] \quad (43)$$

which has the sign of the term between brackets. This term is positive for all I , given that s^u tends to 0 as I tends to infinity and x is strictly concave. It follows that the optimal number of matching places is infinite. Indeed, the planner uses the fact that there are decreasing marginal returns to search investment: he/she opens an infinite number of matching places and sets an arbitrarily small search intensity in each place. Similarly tightness tends to 0. This

result illustrates the inefficiency of the decentralized economy, in which the mass-number of sub-markets is finite. However, it also highlights the asymmetry between the centralized and the decentralized mechanisms. Indeed, the planner can achieve segmentation of the search place without any instrument, while market segmentation in the decentralized economy requires equilibrium wage dispersion.

To obtain a finite mass-number of islands at the social optimum, we can marginally modify the technological side of the model. The point is that for the planner, it is always worth decreasing search intensity, while simultaneously increasing the number of matching places. Therefore, we need to alter the search technology, and in particular the efficiency of effort function. For instance, suppose that $x(s) > 0$ if and only if $s > s_0$. Or, alternatively, suppose that x is strictly convex, and then concave. Subsequently, the optimal number of matching places is finite, and the search intensity is positive in each matching place. The latter simply responds to the following condition familiar to specialists of the efficiency wage literature:

$$s^u(I) x'(s^u(I)) / x(s^u(I)) = 1 \quad (44)$$

Similarly, one can modify the search cost function so that it directly depends on the number of prospected places. This assumption would indeed limit the optimal number of matching places. Of course, the decentralized economy would also be affected by such assumptions. The main difference would be that both the lower bound and the upper bound of the wage distributions would become endogenous. Indeed, sub-markets offering wages such that $V^e(w)$ is close to V^u (i.e. w close to R), or such that $\theta(w)$ is close to 0 (i.e. w close to y) would not be opened in equilibrium. However, the density of the wage offer distribution, as well as the density of the actual wage distribution would still be hump-shaped⁴.

6 Conclusion

This paper offers a search equilibrium model in which firms post wages, and there are homogeneous firms and workers. The main originality of the model relies on the working of the search market. We assume that the search market is segmented by wage, and workers choose the amount of search effort they spend on each (sub-)market. Workers are thus ubiquitous in the sense that they are not bound to choose one and only one market, but can visit the whole set of markets opened in equilibrium. The main result is that a non-degenerate equilibrium wage distribution exists and can replicate two major properties of empirical wage distributions, e.g. the distribution can be single-peaked and right-skewed. All the results are illustrated by a Cobb-Douglas example, in which the wage distribution is a Beta distribution.

A key feature of our model relies on its simplicity. Its main goal is to show that a rather natural extension of the usual directed search assumptions (precisely the consideration of ubiquity in market participation) leads to an equilibrium distribution of wages displaying empirically convincing properties. But, as it stands, our model cannot directly pretend to fit actual wage distributions. Understanding the quantitative role played by ubiquitous search requires the model to be extended in a number of directions. The next step is to introduce heterogeneity. This can be done by abandoning the assumption whereby the productivity of a worker is constant once matched to a firm. One could rather assume that there exists a non-degenerate distribution of output reflecting firm heterogeneity, worker heterogeneity, or both. Another area of study

⁴The model would become more difficult to solve, and thus would lose some of its appealing features.

concerns the profile of wages. A well-documented property of individual wage profile is that it is increasing with tenure. Such a property does not arise in our model but it does occur in models with on-the-job search. Thus, another possible step is to add into our model of ubiquitous search the possibility of searching while employed.

Some policy issues also remain open. Unlike the standard directed search model, the ubiquitous search model ends up with an equilibrium that is not efficient. Hence, other instruments are needed to achieve efficiency.

APPENDIX

A Proof of Lemma 1

We have $\phi(R, R) \geq 0$, $\phi(y, R) \leq 0$. Then, note that according to Assumption A2, the matching function is strictly concave. This implies that $[\theta m(\theta)]' = M_V(1, \theta) > 0$ and $[\theta m(\theta)]'' = M_{VV}(1, \theta) < 0$. As $[\theta m(\theta)]' = m(\theta)[1 - \alpha(\theta)]$, resulting in

$$\begin{aligned} [\theta m(\theta)]'' &= m'(\theta)[1 - \alpha(\theta)] - \alpha'(\theta)m(\theta) \\ &= m(\theta) \left[-\frac{\alpha(\theta)}{\theta}(1 - \alpha(\theta)) - \alpha'(\theta) \right] \end{aligned}$$

It follows that:

$$\alpha'(\theta) > -\frac{\alpha(\theta)}{\theta}(1 - \alpha(\theta)) \quad (45)$$

Now, consider the derivative of function ϕ with respect to w :

$$\phi_w(w, R) = \alpha'(\theta(w))\theta'(w)(y - R) - 1$$

with

$$\alpha(\theta(w))\frac{\theta'(w)}{\theta(w)} = -\frac{1}{y - w} \quad (46)$$

Equations (45) and (46) imply that

$$\phi_w(w, R) < \frac{1 - \alpha(\theta(w))}{y - w}(y - R) - 1 = \frac{-\phi(w, R)}{y - w}$$

This relationship implies that $\phi_w(w, R) < 0$ whenever $\phi(w, R) > 0$. Consequently, the equation $\phi(w, R) = 0$ has a unique root in w .

B Proof of Proposition 1

Part (i). The formulas appearing in (i) simply replicate relations (7) (8), (11) and (12).

Part (ii). As a preliminary step, let us examine the property of the function $\sigma : [z, y] \rightarrow \mathbb{R}$ given by equation (7). Assumption A1 implies that $\sigma(z) = 0$ and $\sigma'(R) > 0$. Solving system (13)-(16) is reduced to finding (w^*, R^*) such that

$$c'[\sigma(R)] = x'[\sigma(R)]\theta(w)m(\theta(w))\frac{w - R}{r + q} \quad (47)$$

$$w = \alpha(\theta(w))y + [1 - \alpha(\theta(w))]R \quad (48)$$

From Lemma 1, equation (48) implicitly defines a unique $w_1 = w_1(R)$. It is strictly increasing in R , with $z < w_1(z) < w_1(y) = y$. Now, consider the following function:

$$J(w, R) = x'[\sigma(R)]\theta(w)m(\theta(w))\frac{w - R}{r + q} - c'[\sigma(R)]$$

An equilibrium solves $K(R) = J(w_1(R), R) = 0$. Assumptions A1 and A2 together with the properties of the function σ established below imply that

$$\begin{aligned} K(z) &= \theta(w_1(z))m(\theta(w_1(z))) \frac{w_1(z) - z}{r+q} \lim_{R \rightarrow z} x'(\sigma(R)) > 0 \\ K(y) &= -c'(\sigma(y)) < 0 \end{aligned}$$

Therefore, it is sufficient to show that function K is strictly decreasing. But,

$$K'(R) = w_1'(R)J_w(w_1(R), R) + J_R(w_1(R), R)$$

Since $J_w(w_1(R), R) = 0$ for all R , we have

$$K'(R) = J_R(w_1(R), R) = \left[x''\theta m(\theta) \frac{w_1(R) - R}{r+q} - c'' \right] \sigma'(R) - x'\theta m(\theta) \frac{1}{r+q} < 0$$

It follows that R^* is unique. Then, $w^* = w_1(R^*)$ and $\theta^* = m^{-1}\left(\frac{h(r+q)}{y-w}\right)$ are uniquely defined.

C Proof of Proposition 2

The formulas appearing in Proposition 2 have been established in the text. It remains to show that they define a unique equilibrium. To do this, one can remark that thanks to Assumption A2 equation (23) defines a unique specific market tightness function $\theta(w)$. Then, equation (24) can be solved in s as a function of w , S and R . Let $e(w, R, S)$ be this unique solution. From Assumption A1 and the implicit function theorem, the partial derivatives of $e(w, R, S)$ are such that:

$$e_S = \frac{c''(S)}{c'(S)} \frac{x'(e)}{x''(e)} < 0, \quad e_R = \frac{1}{w-R} \frac{x'(e)}{x''(e)} < 0$$

In addition, $\lim_{S \rightarrow 0} e(w, R, S) < \infty$, while $\lim_{S \rightarrow \infty} e(w, R, S) = 0$.

Now, substitute $e(w, R, S)$ for $s(w)$ in equation (25), and consider the function ψ such that

$$\psi(R, S) = R - z + c(S) - \int_R^y x[e(w, R, S)]\theta(w)m[\theta(w)] \frac{w-R}{r+q} dw \quad (49)$$

The properties of the function ψ are as follows:

$$\begin{aligned} \lim_{S \rightarrow 0} \psi(R, S) &= R - z - \int_R^y x[e(w, R, 0)]\theta(w)m[\theta(w)] \frac{w-R}{r+q} dw \\ \lim_{S \rightarrow \infty} \psi(R, S) &= +\infty \\ \psi_S(R, S) &= c'(S) \left(1 - \int_R^y e_S dw \right) > 0 \end{aligned}$$

It follows that there exists a unique $S_1 \equiv S_1(R)$ such that $\psi(R, S_1) = 0$ iff $\lim_{S \rightarrow 0} \psi(R, S) \leq 0$.

But,

$$\begin{aligned} \lim_{R \rightarrow z} \psi(R, S) &= c(S) - \int_z^y x[e(w, z, S)]\theta(w)m[\theta(w)] \frac{w-R}{r+q} dw \\ \lim_{R \rightarrow y} \psi(R, S) &= y - z > 0 \\ \psi_R(R, S) &= 1 + \int_R^y \frac{x[s(w)]\theta(w)m(\theta(w))}{r+q} dw - c'(S) \int_R^y e_R dw > 0 \end{aligned} \quad (50)$$

Therefore, there exists a unique $\tilde{R} \in (z, y)$ such that $\lim_{S \rightarrow 0} \psi(R, S) \leq 0$ if and only if $R \leq \tilde{R}$.

To summarize, equation (25) implicitly defines $S_1(R)$ for all $R \in [z, y]$, with

$$S_1(\tilde{R}) = 0 \text{ and } S_1'(R) < \frac{\int_R^y e_R dw}{1 - \int_R^y e_S dw} < 0 \quad (51)$$

Moreover, when $e(w, R, S)$ is substituted for $s(w)$ in equation (26), we obtain another equation defining a unique S as a function of R . We call this function $S_2(R)$. Differentiating this latter equation with respect to R gives:

$$\left(1 - \int_R^y e_S dw\right) \frac{dS}{dR} = -e(R, R, S) + \int_R^y e_R dw$$

Assumption A2 and equation (24) imply that $e(R, R, S) = 0$. Consequently

$$S_2'(R) = \frac{\int_R^y e_R dw}{1 - \int_R^y e_S dw} < 0 \quad (52)$$

So far, we have shown that $S_1'(R) < S_2'(R) < 0$. In addition, $S_2(y) = 0$, which implies that $S_2(\tilde{R}) > S_1(\tilde{R}) = 0$. Lastly, $S_1(z)$ and $S_2(z)$ are given by:

$$S_2(z) = \int_z^y e[w, z, S_2(z)] dw$$

$$c[S_1(z)] = c'[S_1(z)] \int_z^y \frac{x[e(w, z, S_1(z))]}{x'[e(w, z, S_1(z))]} dw \quad (53)$$

Assumption A1 implies that $x(e)/x'(e) > e$ and $c(s)/c'(s) < s$, therefore (53) gives

$$S_1(z) > \frac{c[S_1(z)]}{c'[S_1(z)]} > \int_z^y e[w, z, S_1(z)] dw$$

which proves that $S_1(z) > S_2(z)$. All these properties of the functions $S_1(R)$ and $S_2(R)$ entail that they cross once at a point such that $R^* \in (z, y)$. Thus, the equilibrium values of R and S are unique. It follows that the equilibrium functions $\theta^*(w)$ and $s^*(w)$ given respectively by (23) and (24) are also unique.

D Proof of Proposition 3

Relation (23) and Assumption A2 imply that $\theta(y) = 0$. Therefore, relation (24) and Assumption A1 imply that $s(y) = 0$. Furthermore, (23) shows that $\theta(R)$ is finite and (24) and Assumption A1 then entails that $s(R) = 0$. Since s is continuous and $s(w) > 0$ for all $w \in (R, y)$, the function s is not monotonous. Then, $s'(w)$ is continuous and has the sign of $\phi(w, R)$. The result follows from Lemma 1.

E Proof of Proposition 4

Part (i). According to proposition 3, one has $s(R) = s(y) = 0$. Remembering that $\theta(y) = 0$ and that $\theta(R)$ is finite, relation (29) arrives at $F'(R) = F'(y) = 0$. The result follows from the facts that F' is continuous and $F'(w) > 0$ for all $w \in (R, y)$.

Part (ii). Note that $F''(w)$ has the sign of $\psi(w, R)$. Hence, each root of the equation $\psi(w, R) = 0$ corresponds to a point where $F''(w) = 0$. If Assumption A4 holds, the equation $\psi(w, R) = 0$ has a unique root and the wage offer distribution is \cap -shaped.

Part (iii). Equation (31) shows that:

$$F''(y) = \lim_{w \rightarrow y} \frac{1}{\alpha(\theta(w))} \frac{F'(w)}{y-w} \{[\alpha(\theta(w)) - 1] \gamma(w) - 1\}$$

Using (29) and (23), one arrives at:

$$F''(y) = \frac{u}{vh(r+q)} \lim_{w \rightarrow y} \frac{\theta(w) m(\theta(w)) x(s(w))}{\alpha(\theta(w))} \{[\alpha(\theta(w)) - 1] \gamma(w) - 1\}$$

The result follows from the facts that $\theta(w) m(\theta(w))$ and $x[s(w)]$ are equal to zero when $w \rightarrow y$.

F Proof of Proposition 5

Part (i). As $G'(w) = vF'(w)m[\theta(w)]/\lambda u$, part (i) of Proposition 4 implies that $G'(R) = G'(y) = 0$. The result follows from the fact that $G'(w) > 0$ for all $w \in (R, y)$ and the continuity of G' .

Part (ii). As $m[\theta(w)] = \frac{h(r+q)}{y-w}$ – see (23) –, relation (32) becomes:

$$G'(w) = \frac{v}{\lambda u} \frac{h(r+q)}{y-w} F'(w)$$

Let us denote by w_0 the unique wage such that $\frac{v}{\lambda u} \frac{h(r+q)}{y-w_0} = 1$. A priori, w_0 can be greater or smaller than R . Let us suppose first that $w_0 \geq R$. One has $G'(w) < F'(w)$ for $w < w_0$ and $G'(w) > F'(w)$ for $w > w_0$. Therefore, when $w < w_0$ one has:

$$G(w) = \int_R^w G'(\xi) d\xi < \int_R^w F'(\xi) d\xi = F(w)$$

While, when $w > w_0$, one has:

$$1 - G(w) = \int_w^y G'(\xi) d\xi > \int_w^y F'(\xi) d\xi = 1 - F(w) \quad (54)$$

Hence, one always has $G(w) < F(w)$ when $w_0 \geq R$.

Now, let us assume that $w_0 < R$. Then, one has $G'(w) > F'(w)$ for all $w \geq R$, and (54) holds for all $w \geq R$. Consequently, $G(w) < F(w)$ when $w_0 < R$.

Part (iii). $G''(w)$ has the sign of $\chi(w, R)$. Hence, each root of the equation $\chi(w, R) = 0$ corresponds to a point where $F''(w) = 0$. If Assumption A5 holds, the equation $\chi(w, R) = 0$ has a unique root and the actual wage distribution is \cap -shaped.

G Proof of Proposition 6

Part (i). In the Cobb-Douglas case, equation (31) becomes:

$$\frac{F''(w)}{F'(w)} = \frac{\gamma[\alpha y + (1-\alpha)R - w]}{\alpha(y-w)(w-R)} - \frac{1}{\alpha(y-w)}$$

Integrating this equation with the condition $\int_R^y F'(w)dw = 1$ yields:

$$F'(w) = \frac{(y-w)^{\frac{1-\alpha}{\alpha}\gamma + \frac{1}{\alpha}}(w-R)^\gamma}{\int_R^y (y-\xi)^{\frac{1-\alpha}{\alpha}\gamma + \frac{1}{\alpha}}(\xi-R)^\gamma d\xi}, \forall w \in [0, y]$$

The cdf of the normalized wage satisfies

$$H_F(\omega) = \Pr\left(\frac{w-R}{y-R} \leq \omega\right) = \Pr(w \leq R + \omega(y-R)) = F[R + \omega(y-R)]$$

Therefore, $H'_F(\omega) = (y-R)F'[R + \omega(y-R)]$, and the result follows.

Part (ii). Using the definitions of the functions $\phi(w, R)$ and $\psi(w, R)$, equation (34) becomes:

$$\frac{G''(w)}{G'(w)} = \frac{\gamma(y-R)}{(y-w)(w-R)} - \frac{1+\gamma-\alpha}{\alpha(y-w)}$$

Integrating this equation with the condition $\int_R^y G'(w)dw = 1$ yields

$$G'(w) = \frac{(y-w)^{\frac{(1+\gamma)(1-\alpha)}{\alpha}}(w-R)^\gamma}{\int_R^y (y-\xi)^{\frac{(1+\gamma)(1-\alpha)}{\alpha}}(\xi-R)^\gamma d\xi} \quad (55)$$

The cdf of the actual normalized wage is such that

$$H_G(\omega) = \Pr\left(\frac{w-R}{y-R} \leq \omega\right) = \Pr(w \leq R + \omega(y-R)) = G[R + \omega(y-R)]$$

Therefore one gets $H'_G(\omega) = (y-R)G'[R + \omega(y-R)]$, and the result follows.

H Proof of Proposition 7

Part (i). Let $\rho(i)$ denote the costate variable related to (35). The current-valued Hamiltonian of the problem is:

$$\mathcal{H} = \omega e^{-rt} + \int_0^I \rho(i) \{q[1-u(i)] - u(i)x(s(i))\theta(i)m\theta(i)\} di$$

The first-order conditions are:

$$\frac{\partial \mathcal{H}}{\partial s(i)} = \frac{\partial \mathcal{H}}{\partial \theta(i)} = 0, \quad \frac{\partial \mathcal{H}}{\partial u(i)} = -\dot{\rho}(i)$$

The condition $\partial\mathcal{H}/\partial s(i) = 0$ gives

$$[c'(s(i)) + h\theta(i)x'(s(i))]e^{-rt} + \rho(i)x'(s(i))\theta(i)m(\theta(i)) = 0, \quad \forall i[0, I] \quad (56)$$

Similarly, the condition $\partial\mathcal{H}/\partial\theta(i) = 0$ gives

$$he^{-rt} = -\rho(i)m(\theta(i))[1 - \alpha(\theta(i))], \quad \forall i[0, I] \quad (57)$$

The Euler equations read as:

$$[-y + z - c(s(i)) - h\theta(i)x(s(i))s(i)]e^{-rt} - \rho(i)[q + x(s(i))\theta(i)m(\theta(i))] = -\dot{\rho}(i) \quad (58)$$

Differentiating (57) with respect to time, and taking the result at the stationary state (where $\dot{\theta}(i) = 0$), entails $\dot{\rho}(i) = -r\rho(i)$. Equations (58) and (57) then give

$$\frac{h}{m(\theta(i))} = \frac{[1 - \alpha(\theta(i))][y - z + c(s(i))]}{r + q + x(s(i))\alpha(\theta(i))\theta(i)m(\theta(i))} \quad (59)$$

Similarly, equations (56) and (57) give

$$c'(s(i)) = \frac{\alpha(\theta(i))}{1 - \alpha(\theta(i))}h\theta(i)x'(s(i)) \quad (60)$$

Equations (59) and (60) form a system of two equations with two unknowns $s(i)$ and $\theta(i)$. These conditions are similar to the localized search equilibrium given by the system (17). Therefore a unique solution results and $s(i) = s^l$, while $\theta(i) = \theta^l$, for all $i[0, I]$. Finally, note that the Euler conditions are satisfied for all $u(i) \geq 0$, so that if I were also a control variable, the number of opened matching places would be indeterminate.

Part (ii). Let us denote by ρ the costate variable related to (36), the Hamiltonian of the problem is:

$$\mathcal{H} = \omega e^{-rt} + \rho \left[q(1 - u) - u \int_0^I x(s(i))\theta(i)m(\theta(i)) di \right] \quad (61)$$

The first-order conditions are

$$\frac{\partial\mathcal{H}}{\partial s(i)} = \frac{\partial\mathcal{H}}{\partial\theta(i)} = 0, \quad \forall i[0, I], \quad \frac{\partial\mathcal{H}}{\partial u} = -\dot{\rho} \quad (62)$$

The f.o.c. with respect to $s(i)$ implies

$$\left[c' \left(\int_0^I s(i) di \right) + h\theta(i)x'(s(i)) \right] e^{-rt} + \rho x'(s(i))\theta(i)m(\theta(i)) = 0, \quad \forall i[0, I] \quad (63)$$

Similarly, the f.o.c. with respect to $\theta(i)$ implies

$$he^{-rt} = -\rho m(\theta(i))[1 - \alpha(\theta(i))], \quad \forall i[0, I] \quad (64)$$

The Euler equation reads

$$\left[-y + z - c \left(\int_0^I s(i) di \right) - h \int_0^I \theta(i) x(s(i)) di \right] e^{-rt} - \rho \left[q + \int_0^I x(s(i)) \theta(i) m(\theta(i)) di \right] = -\dot{\rho} \quad (65)$$

Equation (64) shows that $\theta(i)$ does not depend upon i , hence $\theta(i) = \theta^u$ for all $i \in [0, I]$. Then, eliminating ρ between (63) and (64) arrives at:

$$c' \left(\int_0^I s(i) di \right) = \frac{\alpha(\theta)}{1 - \alpha(\theta)} h \theta x'(s(i)) \quad (66)$$

This equation shows that $s(i)$ does not depend on i , hence $s(i) = s^u$, for all $i \in [0, I]$. This last equation gives

$$c'(Is^u) = \frac{\alpha(\theta^u)}{1 - \alpha(\theta^u)} h \theta^u x'(s^u) \quad (67)$$

Differentiating (64) with respect to time, and taking the result at the stationary state (where $\dot{\theta} = 0$), entails $\dot{\rho} = -r\rho$. The Euler equation (65) combined with (64) then gives

$$\frac{h}{m(\theta^u)} = (1 - \alpha(\theta^u)) \frac{y - z + c(Is^u)}{r + q + Ix(s^u)\alpha(\theta^u)\theta^u m(\theta^u)} \quad (68)$$

I Proof that there are no multiple offers

Consider the function $\lambda(w)$ defined over $[R, y]$ such that λ is positive and continuous. Under ubiquitous search, $\lambda(w) = x[s(w)]\theta(w)m[\theta(w)]$. Cut the interval $[R, y]$ into n intervals of the same length $dw = (y - R)/n$. Each interval is an island. On island $i \in \{1, \dots, n\}$, there is a unique wage $w_i = R + (i - 1)dw/n$. Now, consider island i , and cut it into m intervals. Assume that the probability of receiving an offer from any such interval is $\lambda_i dw dt / m$ over the period dt , with $\lambda_i = \lambda(w_i)$.

Let X_i be the number of offers received from island i over the period dt . The probability of receiving $k_i \in \{0, \dots, m\}$ offers is:

$$\Pr(X = k_i) = C_m^{k_i} (\lambda_i dw dt / m)^{k_i} (1 - \lambda_i dw dt / m)^{m - k_i} \quad (69)$$

As $m \rightarrow \infty$, it tends to

$$\Pr(X = k_i) = e^{-\lambda_i dw dt} \frac{(\lambda_i dw dt)^{k_i}}{k_i!} \quad (70)$$

Hence, X_i follows the Poisson law of parameter $\lambda_i dw dt$.

Now, consider the random variable $X = \sum_{i=1}^n X_i$ which is the total number of offers received from all the islands over the period dt . As the different variables are independent draws from Poisson laws, the sum of the draws also follows a Poisson law, whose parameter is the sum of the parameters of the different Poisson laws. Hence,

$$\Pr(X = k) = \Pr\left(\sum_{i=1}^n X_i = k\right) = e^{-\lambda dt} \frac{(\lambda dt)^k}{k!}, \quad \text{with } \lambda = \sum_{i=1}^n \lambda_i dw \quad (71)$$

As $n \rightarrow +\infty$, we obtain that X follows the Poisson law of parameter $\lambda = \lim_{n \rightarrow +\infty} \sum_{i=1}^n \lambda_i dw = \int_R^y \lambda(w) dw$. The remainder of the proof is standard. Following Mortensen (1986),

$$\frac{\Pr(X = k)}{dt} = e^{-\lambda dt} \frac{\lambda^k (dt)^{k-1}}{k!} \quad (72)$$

The right-hand side tends to 0 when dt tends to 0 for all $k > 1$. Similarly, it tends to λ when dt tends to 0 when $k = 1$. It follows that the probability of receiving more than one offer conditional to the fact that the worker receives at least one offer tends to 0 when dt tends to 0.

References

- [1] Acemoglu, D., Shimer, R., 1999. Efficient unemployment insurance. *Journal of Political Economy* 107, 893-928
- [2] Acemoglu, D., Shimer, R., 2000. Wage and technology dispersion. *Review of Economic Studies* 67, 587-607
- [3] Albrecht, J., Gautier, P., Vroman, S., 2005. Equilibrium directed search with multiple applications. *Review of Economic Studies*, forthcoming
- [4] Autor, D., Katz, L.F., 1999. *Handbook of Labor Economics*, vol. 3A, O. Ashenfelter and D. Card eds, 1463-1555
- [5] Burdett, K., Mortensen, D.T., 1998. Wage differentials, employer size, and unemployment. *International Economic Review* 39, 257-273
- [6] Charlot, O., Decreuse, B., Granier, P., 2005. Adaptability, productivity, and educational incentives in a matching model. *European Economic Review* 49, 1007-1032
- [7] Delacroix, A., Shi, S., 2005. Directed search on the job and the wage ladder. *International Economic Review*, forthcoming
- [8] Diamond, P., 1971. A model of price adjustment. *Journal of Economic Theory* 3, 156-168
- [9] Galenianos, M., Kircher, P., 2006. Directed search with multiple applications. Mimeo
- [10] Gautier, P., 2002. Unemployment and search externalities in a model of heterogeneous jobs and workers. *Economica* 273, 21-40
- [11] Hosios, A., 1990. On the efficiency of matching and related models of search and unemployment. *Review of Economic Studies* 57, 279-298
- [12] Julien, B., Kennes, J., King, I., 2006. Ex-post pricing and efficient coordination unemployment. *Canadian Journal of Economics* 38, 174-196
- [13] Moen, E., 1997. Competitive search equilibrium. *Journal of Political Economy* 105, 385-411
- [14] Montgomery, J., 1991. Equilibrium wage dispersion and interindustry wage differentials. *Quarterly Journal of Economics* 106, 163-179
- [15] Moscarini, G., 2001, Excess worker reallocation, *Review of Economic Studies* 68, 593-612
- [16] Moscarini, G., 2005, Job matching and the wage distribution, *Econometrica* 73, 481-516
- [17] Mortensen, D.T., 2000. Equilibrium unemployment with wage-posting: Burdett-Mortensen meet Pissarides. In H. Bunsel, B.J. Christiansen, P. Jensen, N.M. Kiefer and D.T. Mortensen, eds., *Panel data and structural labor market models*, Amsterdam: Elsevier
- [18] Pissarides, C., 2000. *Equilibrium unemployment theory*. MIT Press, 2nd ed

- [19] Postel-Vinay, F., Robin, J.-M., 2002a. The distribution of earnings in an equilibrium search model with state-dependent offers and counteroffers. *International Economic Review* 43, 989-1016
- [20] Postel-Vinay, F., Robin, J.-M., 2002b. Wage dispersion with worker and employer heterogeneity. *Econometrica* 70, 2295-2350
- [21] Stigler, 1961. The Economics of information. *Journal of Political Economy* 69, 213-225
- [22] Van den Berg, G., Ridder, G., 1998. An empirical equilibrium search model of the labor market. *Econometrica* 66, 1183-1221